a calorimeter at the critical point of the hemispherical model gave $H_1$ = 26.5 and $H_2$ = 35 MJ/kg.

The closeness of these results (deviation of not more than 5%) can be considered evidence of the reliability of both enthalpy measurement methods for rarefied infrasonic high temperature flows.

## NOTATION

p, gas pressure; T, gas temperature; h, gas enthalpy; H, total enthalpy; ρ, gas density; u, gas velocity; $c_p$, specific heat of external degrees of freedom; $k_w$, constant for catalytic recombination of atoms on surface; α, heat-exchange coefficient; R, radius of spherical model; Le, Lewis–Semenov number; Re, Reynolds number; r, radius of enthalpy meter channel; w, subscript indicating values on body surface; 0, subscript indicating value in incident flow, as well as initial radius of enthalpy meter channel.

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# USE OF THE IMPROVED THIN-WALL METHOD IN INVESTIGATING HEAT TRANSFER IN A HYPERSONIC WIND TUNNEL

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The authors determine unsteady heat fluxes on a thin-walled model, taking account of two time derivatives of the measured temperature.

A very sensitive method of thermal measurement in investigating heat transfer on models in wind tunnels is the method of microthermocouple measurements on thin-walled models, a method that has found rather wide use [1-3]. Thanks to the use of high-sensitivity measuring equipment [4] and the developed technology of microthermocouple installation [2] high spatial and temporal resolution of temperature fields in the test regions has been achieved [5, 6]. The density of location of microthermocouples on models of wall thickness 0.05-0.1 mm is 7 items per mm, the equipment sensitivity is $1 \cdot 10^{-6}$ V, and the time resolution is $1 \cdot 10^{-4}$ sec.

The data reduction of microthermocouple measurements on thin-walled models is ordinarily performed using the theory of regular regimes [7], or using a general relation connecting the time-dependent heat fluxes q(τ) acting on the model with the rate of change of temperature T(τ) of its internal surface

$$q(\tau) = \rho c \delta \frac{dT(\tau)}{d\tau}. \quad (1)$$

To reduce the error of the approximation (1), we consider relations for calculating unsteady heat fluxes in which we take into account not only the first derivatives of the measured

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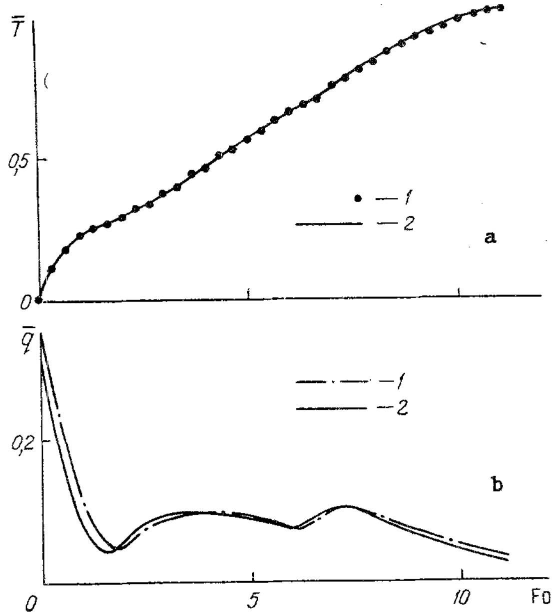


Fig. 1. Dependence of the reduced temperature (a) and the heat flux density (b) on the Fourier number: a) 1) experiment; 2) approximation of the smoothing spline; b) 1) theory with the first derivative; 2) theory with the first two derivatives.

temperature with respect to time, but also the second derivatives.

If we adopt the hypothesis, usual for heat conduction problems, that the temperature field of the model wall is a continuous function and has an infinite number of derivatives with respect to the coordinate x and time $\tau$, then, using the one-dimensional heat-conduction equation and an expansion of $T(x, \tau)$ in a power series in x, we can represent the heat flux function $q(\tau)$ acting on the outer surface of the model wall in the form of the Stefan solution [8]:

$$q(\tau) = \lambda \sum_{n=1}^{\infty} \frac{\delta^{2n-1}}{(2n-1)!\, a^n} T^{(n)}(\tau), \tag{2}$$

where $T(\tau)$ is a function of the temperature of the inner thermally insulated surface of the model wall.

To calculate the time-dependent heat flux using Eq. (2), we need not know the temperature distributions through the model wall thickness. However, the use of Eq. (2) to reduce the experimental data is made difficult by the fact that the problem of differentiating the experimentally determined temperature dependence $T(\tau)$ is an incorrect problem, and that this is particularly important when calculating higher derivatives with respect to $T(\tau)$. Therefore, formulation of the experimental problem is usually limited to cases in which the series on the right side of Eq. (2) converges rapidly and we can retain only the first few terms.

Relation (1), which coincides with the first term of the expansion (2), was obtained by integrating the heat-conduction equation $\rho c\, \partial T/\partial\tau = \lambda\, \partial^2 T/\partial x^2$ with respect to the coordinate x from 0 to $\delta$, and then replacing the average temperature over the model wall thickness

$$T_{av}(\tau) = \frac{1}{\delta} \int_0^{\delta} T(x, \tau)\, dx$$

by the temperature $T(\tau)$ of its inner thermally insulated surface. It is correct to use Eq. (1) when reducing the results of thermal measurements on a thin-walled model because the coefficients in front of the derivatives $T^{(n)}(\tau)$ in Eq. (2) decrease rapidly as n increases, for small enough $\delta$ and large $a$. However, in cases when the characteristic time to propagate heat in the model wall, equal to $\delta^2/(6a)$, is not negligibly small quantity compared with the characteristic time for variation of $T(\tau)$, we must take account at least of the second term of the expansion (2). An expression for the heat flux, more accurate than Eq. (1), has the following form:

$$q(\tau) = \rho c \delta \left[ \frac{dT(\tau)}{d\tau} + \frac{\delta^2}{6a} \frac{d^2 T(\tau)}{d\tau^2} \right]. \tag{3}$$

The quantity $\delta^2/(6a)$ in Eq. (3) describes the delay time for variation of $T(\tau)$ relative to $T_{cp}(\tau)$. This can be seen from the relation

$$T_{av}(\tau) = T\left(\tau + \frac{\delta^2}{6a}\right) + \text{const},$$

which can be derived from the equality

$$\frac{dT_{av}(\tau)}{d\tau} = \sum_{n=1}^{\infty} \frac{\delta^{2n-2}}{(2n-1)!\, a^{n-1}} T^{(n)}(\tau),$$

if we neglect terms of order $O(\delta^k)$, for $k \geq 4$.

The error $q(\tau)$ in Eq. (1) is on the order $\sim O(\delta^3)$, and the error in Eq. (3) is $\sim O(\delta^5)$. Thus, using the thin-walled method of Eq. (3) with improved accuracy we can reduce the main error of the thin-walled method (1), associated with an approximate examination of the heat conduction process through the model wall.

To determine the values $q(\tau)$ we must differentiate the experimental dependence $T(\tau)$. This problem can be solved using matching cubic splines [9] as follows. We minimize the quadratic functional of the second derivative of the cubic spline $S_\Delta(\tau)$ in the region D of the most probable deviations of the values of $S_\Delta(\tau)$ from the measured values of $T(\tau)$:

$$\Phi[S_\Delta(\tau)] = \int_0^{\tau_{max}} [S''_\Delta(\tau)]^2 d\tau \to \min_D \Phi,$$

where

$$D \equiv \{S_\Delta(\tau) : |S_\Delta(\tau) - T(\tau)| < \varepsilon(\tau),\ 0 \leqslant \tau \leqslant \tau_{max}\}.$$

The minimization is accomplished by constructing in region D cubic splines with new nodes and nodal values:

$$\Phi[S_\Delta(\tau)] \to \min_{D_i} \Phi,\quad D_i \equiv \{S_\Delta(\tau) : |S_\Delta(\tau_i) - T(\tau_i)| < \varepsilon(\tau_i),\quad i = 1, \ldots, n\};$$

$$\Phi[S_\Delta(\tau)] \to \min_\Delta \Phi,\quad |S_\Delta(\tau_i) - T(\tau_i)| < \varepsilon(\tau_i),\ i = 1, \ldots, n,$$

where $\tau_i$, $i = 1, \ldots, n$ are the nodes of the original mesh division. The confidence intervals $\varepsilon(\tau_i)$ describing the ranges of allowable deviations of values of the smoothing spline $S_\Delta(\tau_i)$ from the measured values of temperature $T(\tau_i)$ are determined from the primary reduction of the experimental data using the methods of mathematical statistics.

To confirm this method of determining unsteady heat fluxes on a thin-walled model we used the results of the thermal experiment of [5] in a hypersonic pulsed wind tunnel. The model, made in the form of a blunt semicone of vertex angle 30° was set up in the test section in such a way that its planar surface was located parallel to the velocity vector of the unperturbed air flow. The central part of the planar surface had microthermocouples welded to the inner thermally-insulated surface of nichrome foil of thickness $\delta = 0.1$ mm, by the method described in [2].

The experimental results and the results of reducing them are shown in Fig. 1 in the dimensionless coordinates $\bar{T} = (T - T_{in})/(T_{max} - T_{in})$, $\bar{q} = q\delta/[\lambda(T_{max} - T_{in})]$, $Fo = a\tau/\delta^2$. The curves of $\bar{q}(Fo)$ obtained with the aid of Eqs. (1) and (3) are similar, i.e., their behavior agrees qualitatively. However, in the initial period of model heating, corresponding to the unsteady regime of operation of the wind tunnel, there is a quantitative difference that reaches 20% of the range of variation of $\bar{q}(Fo)$. From the readings of the reference microthermocouples, using the method described in [6], we obtained an estimate of the error arising from the nonuniformity of the process of heat propagation in the model wall. This error in the experiment did not exceed 2% of the heat flux values determined.

We shall give a possible explanation of the behavior of the heat flux function $\bar{q}$ as a function of the number Fo, by comparing it with the known wave diagram of operation of the wind tunnel and with the results of experiment [10]. The start-up process in the pulse regime of operation of the wind tunnel is the process of unsteady expansion of the gas and is accompanied by substantial redistribution of energy between its different layers [11]. In the experiment the leading layer of gas, located immediately behind the bow shock, evidently has the stagnation temperature $T_0$, appreciably higher than the gas temperature in the tunnel chamber [10]. For this reason in the initial period of model heating (for Fo $\lesssim$ 1) higher values of heat flux $\bar{q}$ are observed. (The possibility of recording these depends on the width of the region of increased values of $T_0$, i.e., on the degree of rarefaction of the remaining gas in the working part of the tunnel for a given pressure in the chamber.) In accordance with the law of energy conservation in the gas there must exist a region with a decreased value of stagnation temperature $T_0$. This region is characterized by low values of the heat flux $\bar{q}$ (for 1 $\lesssim$ Fo $\lesssim$ 2.5). In the quasisteady regime of operation of the tunnel (for 2.5 $\lesssim$ Fo $\lesssim$ 8) the values of $\bar{q}$ do not change as much. However, beginning at a certain time (for Fo $\gtrsim$ 8) the heat fluxes $\bar{q}$ decrease sharply because of breakdown of the quasisteady flow when it is acted on by the arrival of the reflected rarefaction wave.

It can be seen from Fig. 1 that the method used for reducing the experiment, Eq. (3), taking into account two time derivatives of the measured temperature, in comparison with the ordinary thin-walled method of Eq. (1), gives noticeably increased accuracy of calculating the heat fluxes in the unsteady regime of operation of the hypersonic wind tunnel.

## NOTATION

T, temperature; $T_{in}$, initial temperature; $T_{max}$, maximum temperature; $T_{av}$, average temperature; $\bar{T}$, dimensionless temperature; q, heat flux density; $\bar{q}$, dimensionless heat flux density; S, cubic spline; D, region of smoothing; x, space coordinate; $\tau$, time; $\tau_{max}$, maximum time; Fo, Fourier number; $a$, thermal diffusivity; $\lambda$, thermal conductivity; $\rho$, density; c, specific heat; $\delta$, model wall thickness; $\Delta$, mesh size; i, mesh node number.

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